

39th International Mathematical Olympiad

Taipei, Taiwan

Day I

July 15, 1998

1. In the convex quadrilateral $ABCD$, the diagonals AC and BD are perpendicular and the opposite sides AB and DC are not parallel. Suppose that the point P , where the perpendicular bisectors of AB and DC meet, is inside $ABCD$. Prove that $ABCD$ is a cyclic quadrilateral if and only if the triangles ABP and CDP have equal areas.
2. In a competition, there are a contestants and b judges, where $b \geq 3$ is an odd integer. Each judge rates each contestant as either “pass” or “fail”. Suppose k is a number such that, for any two judges, their ratings coincide for at most k contestants. Prove that $k/a \geq (b - 1)/(2b)$.
3. For any positive integer n , let $d(n)$ denote the number of positive divisors of n (including 1 and n itself). Determine all positive integers k such that $d(n^2)/d(n) = k$ for some n .

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4. Determine all pairs (a, b) of positive integers such that $ab^2 + b + 7$ divides $a^2b + a + b$.
5. Let I be the incenter of triangle ABC . Let the incircle of ABC touch the sides BC , CA , and AB at K , L , and M , respectively. The line through B parallel to MK meets the lines LM and LK at R and S , respectively. Prove that angle RIS is acute.
6. Consider all functions f from the set N of all positive integers into itself satisfying $f(t^2 f(s)) = s(f(t))^2$ for all s and t in N . Determine the least possible value of $f(1998)$.